Wear Detection in Gear System Using Hilbert-Huang Transform

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Fourier methods are not generally an appropriate approach in the investigation of faults signals with transient components. This work presents the application of a new signal processing technique, the Hilbert-Huang transform and its marginal spectrum, in analysis of vibration signals and faults diagnosis of gear. The Empirical mode decomposition (EMD), Hilbert-Huang transform (HHT) and marginal spectrum are introduced. Firstly, the vibration signals are separated into several intrinsic mode functions (IMFs) using EMD. Then the marginal spectrum of each IMF can be obtained. According to the marginal spectrum, the wear fault of the gear can be detected and faults patterns can be identified. The results show that the proposed method may provide not only an increase in the spectral resolution but also reliability for the faults diagnosis of the gear.

Key Words : Wear, Gear, Faults Diagnosis, Empirical Mode Decomposition, Hilbert-Huang Transform

1. Introduction

Gear mechanisms are an important element in a variety of industrial applications such as machine tool and gearbox. An unexpected failure of the gear mechanism may cause significant economic losses. For that reason, faults detection in gears has been the subject of intensive research. Vibration signal analysis has been widely used in the faults detection of rotation machinery. Many methods based on vibration signal analysis have been developed. These methods include power spectrum estimation, cepstrum analysis, synchronous time average and phase demodulation, which have been proved to be effective in gear fault detection. However, these methods are based on the assumption of stationary and linearly of the vibration signal. Therefore, new techniques are needed to analyze vibration for faults detection in gear mechanism. Gear faults by their nature are time localized transient events. To deal with non-stationary and non-linearity signals, time-frequency analysis techniques such as the Short Time Fourier Transform (STFT) (Cohen, 1995), Wavelet Transform (WT) (Lin and Qu, 2000 ; Oh et al., 2004 ; Yang et al., 2005 ; Lee et al., 2004 ; Wang and Mcfadden, 1995) and Wigner-Ville distribution (WVD) (Staszewski et al., 1997; Kim et al., 2005a; 2005b ; Matz and Hlawatsch, 2003 ; Hlawatsch and Kozek, 1993) are widely used. The STFT (Cohen, 1995) uses sliding windows in time to capture the frequency characteristics as functions of time. Therefore, spectrum is generated at discrete time instants. An inherent drawback with the STFT is the limitation between time and frequency resolutions. A finer frequency resolution can only be achieved at the expense of time re-

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solution and vice-versa. Furthermore, this method requires large amounts of computation and storage for display. The Wavelet Transform (WT) has been successfully used in non-stationary vibration signal processing and fault detection. WT is capable of providing both time-domain information and frequency-domain information simultaneously. A very appealing feature of the wavelet analysis is that it provides a uniform resolution for all the scales. Limited by the size of the basic wavelet function, the downside of the uniform resolution is uniformly poor resolution. Moreover, an important limitation of the wavelet analysis is its non-adaptive nature. Once the basic wavelet is selected, one will have to use it to analyze all the data. This leads to a subjective assumption on the characteristic of the analyzed signal. As a consequence, only signals feature that correlate well with the shape of the wavelet function have a chance to lead to coefficients of high value. All other feature will be masked or completely ignored. The Wigner-Ville distribution (WVD) is a basic time-frequency representation, which is part of the Cohen class of distribution. Furthermore, it possesses a great number of good properties and is of popular interest for non-stationary signal analysis. Therefore, the Wigner-Ville distribution has received considerable attention in recent years as an analysis tool for non-stationary or time-varying signals. It has been widely used in the areas of structure-bone noise identification (Wahl and Bolton, 1993), optics (Bartelt et al., 1980), machinery condition monitoring (Meng and Qu, 1991 ; Leuridan and Auweraer, 1994 ; Shin and Jeon, 1993) and so on. The difficulty with this method is the severe cross terms as indicated by the existence of negative power for some frequency ranges.

In this work, we introduce a novel approach for nonlinear, non-stationary data analysis. Application of Hilbert-Huang transform method to fault diagnosis of gear wear is presented. The methodology developed in this paper decomposes the original times series data in intrinsic oscillation modes, using the empirical mode decomposition. Then the Hilbert transform is applied to each intrinsic mode function. Therefore, the time-frequency distribution is obtained. The basic method is introduced in detail. The Hilbert-Huang transform is applied in the research of the faults diagnosis of the gear wear. The experimental results show that the presented method cans effectively diagnosis the faults of gear wear.

This paper has been organized as follows: Section 1, gives a brief introduction of the time-frequency analysis technology. Section 2, gives a brief description of the Hilbert-Huang transformation (HHT). Section 3, presents the method and procedure of the fault diagnosis based on HHT. Section 4, gives the applications of the method based on marginal spectrum and instantaneous energy spectrum to faults diagnosis of gear wear. Section 5, gives the main conclusions of this paper.

2. Introduction of the Hilbert-Huang Transformation (Huang, W. et al.,1998)

Hilbert-Huang transformation is an emerging novel technique of signal decomposition having many interesting properties. In particular, HHT has been applied to numerous scientific investigations, such as biomedical signals processing (Huang, W. et al., 1998a ; 1998b ; 1999), geophysics (Huang, N. E. et al., 1999 ; Wang et al., 1999 ; Wu et al., 1999 ; Datig and Schlurmann, 2004), image processing (Nunes et al., 2003), structural testing (Quek et al., 2003), faults diagnosis (Loutridis, 2004), nuclear physics (Montesinos et al., 2003) and so on. In order to facilitate the reading of this paper we will introduce in detail the Hilbert-Huang transformation, which is a relatively novel technique.

2.1 The concept of intrinsic mode function

Huang et al.(1998) have defined IMFs as a class of functions that satisfy two conditions:

(1) In the whole data set, the number of extrema and the number of zero-crossings must be either equal or differ at most by one;

(2) At any point, the mean value of the envelope defined by the local maxima and the enve-

lope defined by the local minima is zero.

2.2 Empirical mode decomposition method: the sifting process

Empirical mode decomposition method is developed from the simple assumption that any signal consists of different simple intrinsic mode oscillations. The essence of the method is to identify the intrinsic oscillatory modes (IMFs) by their characteristic times scales in the signal and then decompose the signal accordingly. The characteristics time scale is defined by the time lapse between the successive extremes.

To extract the IMF from a given data set, the sifting process is implemented as follows. First, identify all the local extrema, and then connect all of the local maxima by a cubic spline line as the upper envelope. Then, repeat the procedure for the local minima to produce the lower envelope. The upper and lower envelopes should cover all the data between them. Their mean is designated $m_1(t)$, and the difference between the data and $m_1(t)$ is $h_1(t)$, i.e.:

$$x(t) - m_1(t) = h_1(t) \tag{1}$$

Ideally, $h_1(t)$ should be an IMF, for the construction of $h_1(t)$ described above should have forced the result to satisfy all the definitions of an IMF by construction. To check if $h_1(t)$ is an IMF, we demand the following conditions: (i) $h_1(t)$ should be free of riding waves i.e. the first component should not display under-shots or over-shots riding on the data and producing local extremes without zero crossing. (ii) To display symmetry of the upper and lower envelops with respect to zero. (iii) Obviously the number of zero crossing and extremes should be the same in both functions.

The sifting process has to be repeated as many times as it is required to reduce the extracted signal to an IMF. In the subsequent sifting process steps, $h_1(t)$ is treated as the data; then:

$$h_1(t) - m_{11}(t) = h_{11}(t) \tag{2}$$

Where $m_{11}(t)$ is the mean of the upper and lower envelops of $h_1(t)$.

This process can be repeated up to k times; $h_{1k}(t)$ is then given by

$$h_{1(k-1)}(t) - m_{1k}(t) = h_{1k}(t) \tag{3}$$

After each processing step, checking must be done on whether the number of zero crossings equals the number of extrema.

The resulting time series is the first IMF, and then it is designated as $c_1(t) = h_{1k}(t)$. The first IMF component from the data contains the highest oscillation frequencies found in the original data $x(t)$.

This first IMF is subtracted from the original data, and this difference, is called a residue $r_1(t)$ by:

$$x(t) - c_1(t) = r_1(t) \tag{4}$$

The residue $r_1(t)$ is taken as if it was the original data and we apply to it again the sifting process. The process of finding more intrinsic modes $c_i(t)$ continues until the last mode is found. The final residue will be a constant or a monotonic function; in this last case it will be the general trend of the data.

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{5}$$

Thus, one achieves a decomposition of the data into n-empirical IMF modes, plus a residue, $r_n(t)$, which can be either the mean trend or a constant.

2.3 The hilbert-huang transform (HHT) and its spectrum

Having obtained the IMFs using EMD method, one applies the Hilbert transform to each IMF component.

$$H[c_i(t)] = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{c_i(\tau)}{t-\tau} d\tau \tag{6}$$

With this definition $c_i(t)$ and $H[c_i(t)]$ form a complex conjugate pair, which defines an analytic signal $z_i(t)$:

$$z_i(t) = c_i(t) + jH[c_i(t)] \tag{7}$$

Which can be expressed as

$$z_i(t) = a_i(t)\exp(j\omega_i(t)) \tag{8}$$

With amplitude $a_i(t)$ and phase $\theta_i(t)$ defined by the expressions:

$$a_i(t)=\sqrt{c_i^2(t)+H^2[c_i(t)]} \quad (9)$$

$$\theta_i(t)=\arctan\left(\frac{H[c_i(t)]}{c_i(t)}\right) \quad (10)$$

Therefore, the instantaneous frequency $\omega_i(t)$ can be given by:

$$\omega_i(t)=\frac{d\theta_i(t)}{dt} \quad (11)$$

Thus the original data can be expressed in the following form:

$$x(t)=\mathrm{Re}\sum_{i=1}^{n}a_i(t)\exp\left(j\int\omega_i(t)\,dt\right) \quad (12)$$

where the residue $r_n(t)$ has been left out. Re $\{\cdot\}$ denotes the real part of a complex quantity.

Eq. (8) enables us to represent the amplitude and the instantaneous frequency, in a three-dimensional plot, in which the amplitude is the height in the time-frequency plane. This time-frequency distribution is designated as the Hilbert-Huang spectrum $H(\omega,\ t)$:

$$H(\omega,t)=\mathrm{Re}\sum_{i=1}^{n}a_i(t)\exp\left(j\int\omega_i(t)\,dt\right) \quad (13)$$

With the Hilbert-Huang spectrum defined, the marginal spectrum, $h(\omega)$, can be defined as

$$h(\omega)=\int_0^T H(\omega,t)\,dt \quad (14)$$

where T is the total data length.

The Hilbert spectrum offers a measure of amplitude contribution from each frequency and time, while the marginal spectrum offers a measure of the total amplitude contribution from each frequency.

Therefore, local marginal spectrum of each IMF component is given as

$$h_i(\omega)=\int_0^T H_i(\omega,t)\,dt \quad (15)$$

The local marginal $h_i(\omega)$ spectrum offers a measure of the total amplitude contribution from the frequency that we are interested in.

In particular, the instantaneous energy spectrum $E_i(t)$ of mode $c_i(t)$ will be used defined as

$$E_i(t)=\int_{-\infty}^{+\infty}H_i^2(\omega,t)\,d\omega \quad (16)$$

The instantaneous energy spectrum $E_i(t)$ provides information about the variation of energy respect to time for the mode $c_i(t)$.

3. Proposed HHT Method for Faults Detection of Gear Wear

The procedure of proposed marginal spectrum method is given as follows:

(1) To decompose the vibration signal $x(t)$ using EMD and to obtain IMFs;

(2) To calculate the marginal spectrum $h_i(\omega)$ according to Eq. (15);

(3) To calculate the instantaneous energy spectrum $E_i(t)$ according to Eq. (16);

(4) To draw a diagnostic conclusion according to the marginal spectrum $h_i(\omega)$ and instantaneous energy spectrum $E_i(t)$.

4. Application of HHT to Fault Detection of Gear Wear

The gear test apparatus used in this study is shown in Fig. 1. The vibration signals of the gear wear are sampled on a single-stage gearbox. A pair of spur gears, with the module of 2.5 mm, is tested. Localized wear defect of the driven gear has a chipped tooth, from zero thickness at pitch point to 25% thichness at the tooth top, to simulate serious wear. The driving gear has 28 teeth and the driven gear has 36 teeth. Thertfore, the transmission ratio is 36/28, which means that an decrease in rotation speed is achieved. The input shaft is driven by an AC motor. The input speed of the spindle is 1473 r/min, that is, the rotating frequency of the output shaft f_r is 19.11 Hz. Therefore, the tooth-meshing frequency is 688 Hz. The monitoring and diagnostic system is composed of four accelerometers, amplifiers, B&K 3560 spectrum analyzer and a computer. The sampling span is 6.4 kHz, the sampling frequency is 16384 Hz and the sampling point is 2048.

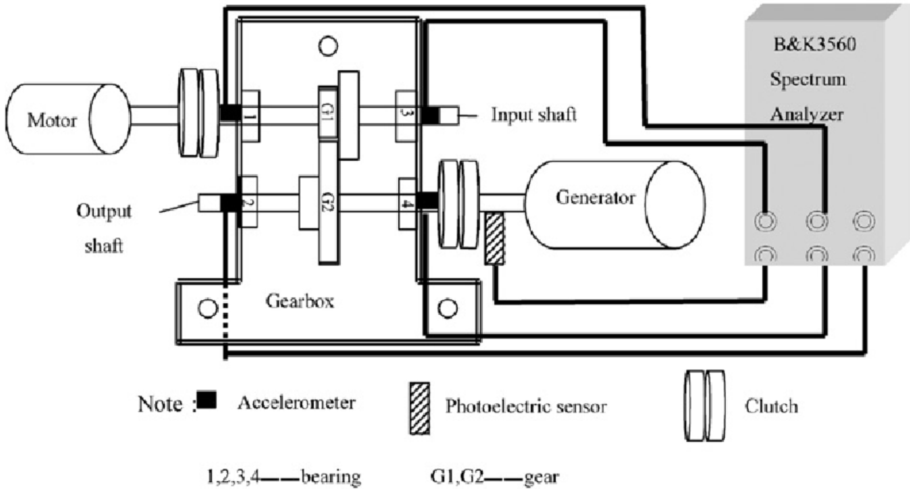

Fig. 1 The gearbox and monitoring system

It is well known that the most important components in gear vibration spectra are the tooth-meshing frequency and its harmonics, together with sidebands due to modulation phenomena. The increment in the number and amplitude of such sidebands may indicate a fault condition. Moreover, the spacing of the sidebands is related to their source. In particular, faults localized on one tooth or a few teeth, such as gear crack or gear wear, produce modulation effects only during the engagement of the faulted teeth, but repeated once each revolution of the gear. As a consequence, the spectrum presents a large number of sidebands of the tooth-meshing frequency and its harmonics, spread over a wide frequency range, spaced by the rotation frequency of the faulted gear and characterized by low amplitude (Randall, 1982; Dalpiaz et al., 2000).

The original vibration signal with gear wear is displayed in Fig. 2. It is clear that there are periodic impacts in the vibration signal. There are significant fluctuations in the peak amplitude of the signal. However, it is hardly possible to evaluate the gear fault condition only through such time domain vibration signal.

Fig. 3 shows the power spectrum of the vibration signal with gear wear. 1376 Hz frequency component, which is the second harmonic of the meshing frequency, can be clearly seen in Fig. 3. But there is no fault frequency component around

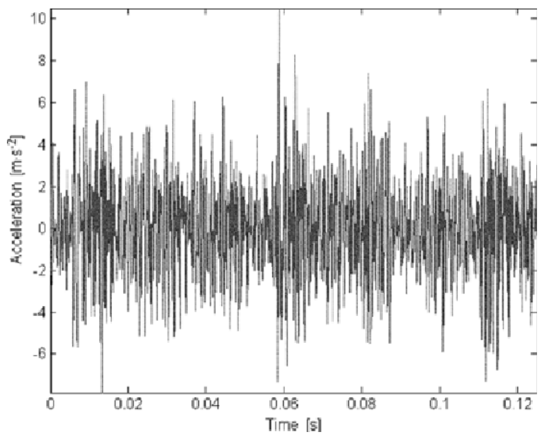

Fig. 2 Time-domain vibration signal with gear wear

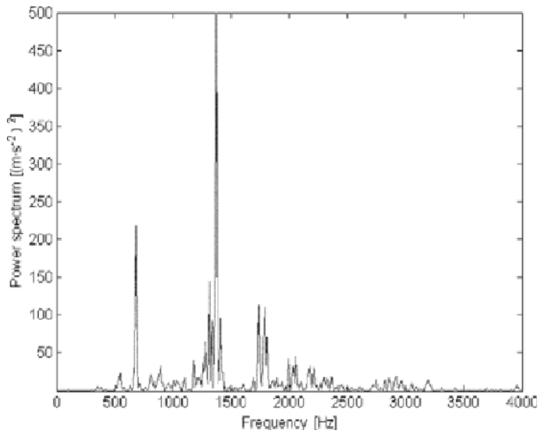

Fig. 3 Power spectrum of the vibration signal

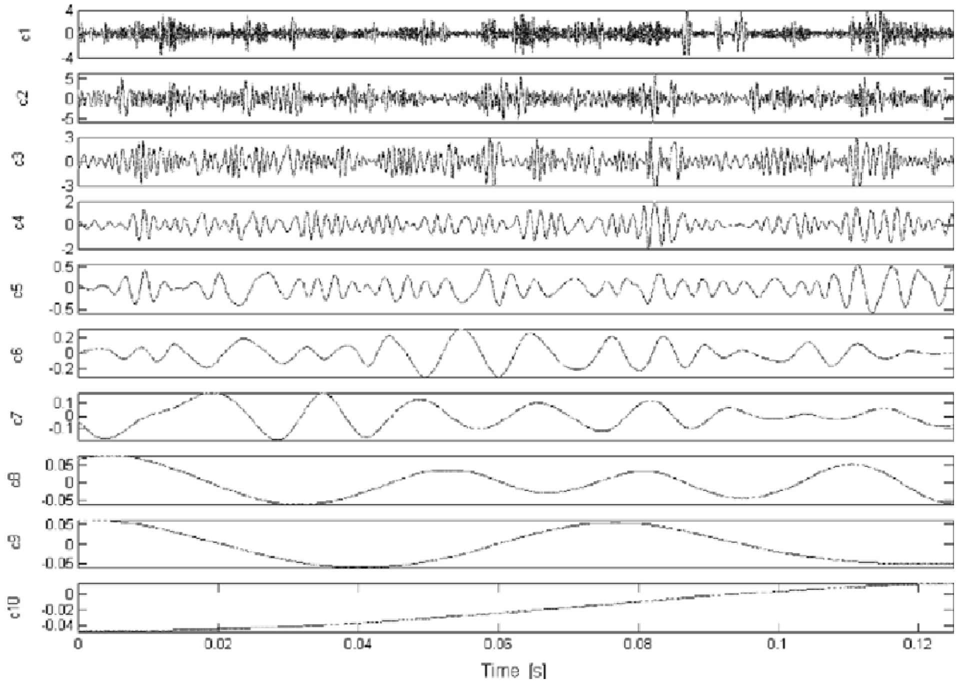

Fig. 4 IMFs of the vibration signal shown in Fig. 2

19.11 Hz. Therefore, classical Fourier analysis has some limitation such as unable to process non-stationary signals.

To the data of Fig. 2, the EMD algorithm is applied. Fig. 4 displays the empirical mode decomposition in ten IMFs of the vibration signal in Fig. 2. The decomposition identifies ten modes: $c_1 \sim c_9$ represents the frequency components excited by the gear wear defects, c_{10} is the residue, respectively. Mode c_1 contains the highest signal frequencies, mode c_2 the next higher frequency band and so on.

From Fig. 4, it can be easily proven that the EMD decomposes vibration signal very effectively on an adaptive method. The Hilbert-Huang transform can be applied to each IMF $c_i(t)$, resulting in marginal spectrum $h_i(\omega)$. The marginal spectrum $h_i(\omega)$ is shown in Fig. 5. The mode c_1 with marginal spectrum centered from 1000 Hz to 7500 Hz and mode c_2 with marginal spectrum centered from 600 Hz to 3500 Hz. Therefore, it is can be concluded that mode c_1 and mode c_2 are the high frequency noise. The mode c_3 with marginal spectrum centered at 1376 Hz, which can be obviously associated with the second harmonic of

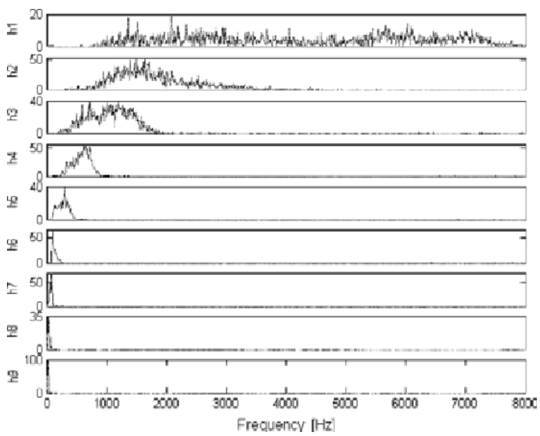

Fig. 5 Marginal spectrum of IMFs

the meshing frequency. Mode c_4, c_5, c_6 and c_7 are associated with the high harmonic of the rotational frequency of the output shaft. The mode c_8 with marginal spectrum centered at 38.22 Hz, which can be obviously associated with the second harmonic of the rotational frequency of the output shaft and modewith the rotational frequency of the output shaft itself (19.11 Hz). Moreover, the amplitude of the marginal spectrum h_9 is larger than that of the others. So it can be

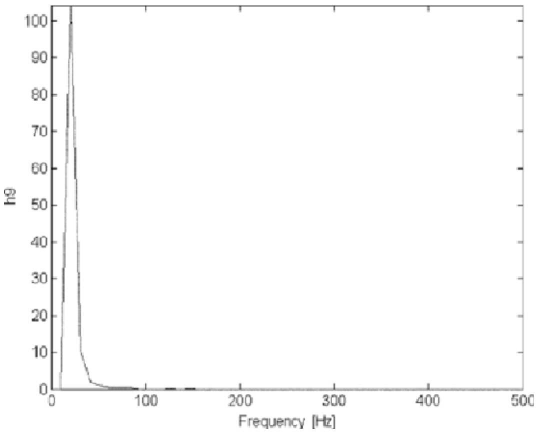

Fig. 6 Marginal spectrum h_9

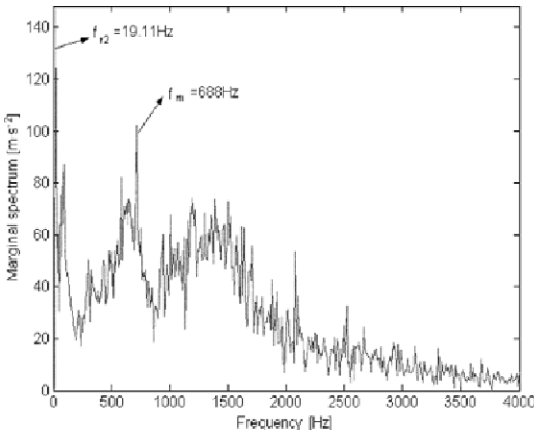

Fig. 7 Marginal spectrum of vibration signal shown in Fig. 2

concluded that the wear faults is occurred in the gear of output shaft. Therefore, it seems that mode c_9 best represents the time scale of the transient caused by gear wear damage. In other words, the IMF mode c_9 is related to gear wear defect of the output shaft. Fig. 6 is the zoomed figure of the marginal spectrum h_9. The total marginal spectrum is shown in Fig. 7. From Fig. 7, the rotational frequency of the output shaft (19.11 Hz) and the tooth-meshing frequency (688 Hz) can be easily seen.

The instantaneous energy $IE(t)$ is shown in Fig. 8. The presences of gear wear fault results in a sudden increase of vibration energy. In Fig. 8, the instantaneous energy is relatively high and has the period impulse associated with period of

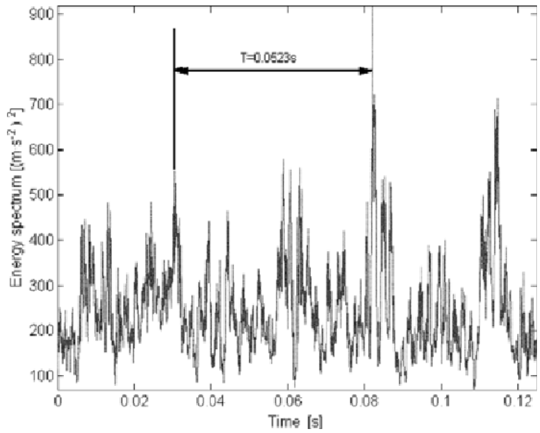

Fig. 8 Instantaneous energy of the signal shown in Fig. 2

the output shaft revolution.

5. Conclusions

In this paper, a method for fault diagnosis of gear wear is presented based on a newly developed signal processing technique named as Hilbert-Huang transform. Using EMD method, the original vibration signals of wear faults can be decomposed into intrinsic modes. Therefore, we can recognize the vibration modes that coexist in the system, and to have a better understanding of the nature of the fault information contained in the vibration signal. According to marginal spectrum, the characteristic frequency of the gear wear faults can be easily recognized. Practical vibration signal monitored from a gearbox with gear wear fault is analyzed by the presented method. The experimental result has been shown that Hilbert-Huang transform can be used as an effective diagnostic method for gear wear faults.

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